
Conflict Escalation, Signaling and Screening Compatibility or Competition?

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What is the cause of war under conditions of asymmetric information? Some argue that war is caused by states' failure in deterrence of preemptive attacks, while others argue that it is caused by states' fear of other states' intention. By focusing on how states exchange their *resolves* to resort to military forces designed to prevent preemptive attacks, this study explores the causes of war under conditions of asymmetric information.

States need to deter its neighbors from aggressive actions; however, they also need to relieve them at the same time. If a state puts too much stress on deterrence, it may frighten the neighbors, which encourages them to attack preemptively. Thus, it is difficult for all states to declare their intentions in a proper manner under conditions of asymmetric information.

The Gulf War in 1990-91 provides a good example of the central issues. Before invading Kuwait, Iraqi President Saddam Hussein made a variety of demands of Kuwait.¹ However, neither Kuwait nor the United States ever tried to determine the concession point at which Saddam would have been satisfied by making increasingly generous offers.² As a result, they failed to deter Saddam's invasion. In this sense, the war was caused by their failure of *deterrence*.

In the meantime, Washington also failed to reassure Saddam through the signaling of benign intentions. In those days, Saddam regarded America as an imperialist

power bent on economic warfare against the Arab world. Washington, however, sent ineffective signals that increased Saddam's suspicion of the United States' intention³. As a result, the United States failed to reassure Saddam and encouraged him to invade Kuwait. In this sense, the war was caused by their failure of *reassurance*.

Today, states have developed a variety of techniques to overcome asymmetric information. Some might declare their determination by taking costly actions such as arms buildups, while some might test their neighbors' limits by making proposals. The most important point of those techniques is to interchange their *resolution* with each other in a proper manner. This paper seeks to clarify under what condition states can transmit their resolves effectively and address the problem, and under what condition they fail in transmission and war is likely.

This article proceeds as follows. First, I review previous studies and their deficiencies, and propose ways of correcting these defects. Second, I present a basic model that makes it possible to consider the compatibility of both signaling and screening. Based on this model, I clarify several findings regarding when war is more likely and when it is less likely.

THEORETICAL BACKGROUND

Prevention of the Preemptive Attacks

In existing literatures on the causes of war, many have argued that the main cause of war is the gap between distribution of power and distribution of benefit, meaning that the greater the disparity between distribution of power and distribution of benefit, the more likely war will occur.⁴ If a

³ *Ibid*, 161-65.

⁴ Robert Powell, *In the Shadow of Power: States and Strategies in International Politics* (Princeton: Princeton University Press, 1999), 6; Suzanne Werner, "The Precarious Nature of Peace: Resolving the Issues, Enforcing the Settlement, and Renegotiation the Terms,"

¹ Janice Gross Stein, "Deterrence and Compellence in the Gulf, 1990-91: – A Failed or Impossible Task?," *International Security*, 17 (1992): 147-7.

² *Ibid*, 159-60.

state feels that it is not getting its fair share of benefits, it has an incentive to change the status quo, and that includes use of force.

The relevant diplomatic policies for the prevention of such calamities combine Deterrence and Reassurance.⁵ Deterrence here is defined as the use of threats to dissuade an opponent from undertaking damaging actions in the future.⁶ The origin of bargaining theory in international relations can be found in James Fearon's *Rationalist Explanations for War*.⁷ Assuming that a state's resolve is based in asymmetric information, Fearon has attempted to elaborate the causes of war by observing whether it can send proper signals to deter other states.⁸ Thus, war could be regarded as the failure of deterrence.

In contrast, Reassurance is defined as the communication of benign and unaggressive intentions by the defender to the prospective challenger.⁹ Hence, deterrence is designed to use aggressive intentions to dissuade the opponent from preemptive attack, while reassurance relies on the demonstration of benign intentions. Robert Powell's *In the Shadow of Power* developed the theory by starting with the rigid assumption that the gap between the distribution of benefits and the distribution of power causes war.¹⁰ Powell thus argued that the failure of

concessions that states make to other states is the decisive cause of war. If states succeed in making concessions to other states properly, they can settle disputes. In other words, the failure of reassurance could cause a war.

Asymmetric information

Then, why do states fail to take those measures against preemptive attacks? The main reason is that every state has an incentive to misrepresent its resolve under conditions of asymmetric information.¹¹ Hence, every state questions the credibility of other states' discourses, and needs a way to make their messages credible in order to reach agreements with other states and avoid unnecessary wars becomes paramount. There are two potential means of making messages credible: *Signaling* and *Screening*. Signaling is defined as taking action to make threats to use force credible.¹² For example, if a state leader decides to expand troop numbers or to declare publicly that the state will resist if attacked, it can declare its resolution more strongly to other states.¹³

Screening is defined as the identification of how much degree an opponent is resolved through the use of increasingly generous proposals that facilitate settling early with the weakly resolved types.¹⁴ For example, if a state suggests a rigorous

American Journal of Political Science, 43 (1999): 929.

⁵ Richard Lebow, "Deterrence and Reassurance: Lessons from the Cold War," *Global Dialogue*, 3 (2001): 128.

⁶ Alexander George, *Forceful Persuasion: Coercive Diplomacy as an Alternative to War* (Washington, DC: United States Institute of Peace Press, 1991), 5.

⁷ James D. Fearon, "Rationalist Explanations for War," *International Organization* 49 (3), 379-414.

⁸ James D. Fearon, "Signaling Foreign Policy Interests: Tying Hands versus Sinking Costs," *Journal of Conflict Resolution* 41 (1), 68-90.

⁹ Lebow, "Deterrence and Reassurance: Lessons from the Cold War," 432.

¹⁰ Powell, *In the Shadow of Power: States and Strategies in International Politics*, 86 – 104.

¹¹ Fearon, "Rationalist Explanations for War," *International Organization*, 379- 414. Recent studies have tended to emphasize the *Commitment Problem* as an explanatory cause of war. However, the analysis of that factor requires a firm basis in Fearon's first explanatory variable, *Asymmetric Information*, which I thus focus on as the cause of war in this study.

¹² Fearon, "Signaling Foreign Policy Interests: Tying Hands Versus Sinking Costs," 69.

¹³ James D. Fearon, "Domestic Political Audiences and the Escalation of International Disputes," *American Political Science Review*, 88 (1994), 577- 92.

¹⁴ Scott Wolford, Dan Reiter, and Clifford Carrubba, "Information, Commitment, and War," *Journal of Conflict Resolution*, 55 (2011), 567.

proposal on some issue to an opponent and the opponent accepts it quickly, it can be assumed that an opponent is not resolved so much to fight over the issue.

Therefore, in this section, two main points could be drawn: First, states fail either in deterrence or reassurance due to asymmetric information. Second, states overcome asymmetric information by means of signaling and screening.

Table 1. Diplomatic Policy and its Techniques

In previous studies, many scholars

	Signaling	Screening
Deterrence	Fearon (1997)	x
Reassurance	x	Powell (1999)

have looked into conditions and success factors for either signaling or screening separately, but not together. However, wars have occurred because these two techniques have not been compatible.¹⁵ This study assumes that it is important to examine the *compatibility* of signaling and screening and to find the conditions under which we can achieve both at the same time, and thus clarify the cause of war.

Convergence of States' Beliefs

How can previous research be modified in order to address the problem? As mentioned, signaling and screening need to be examined in combination in order to analyze the causes of war. In addition to Powell's, Slantchev argues that even though there is a great disparity in capability among states, a weaker state may threaten to provoke a war in order to induce a stronger state to offer better terms.¹⁶ Therefore, expectations need to converge not only with respect to the distribution of power, but also the resolve of

states, in order to prevent preemptive attacks from weaker actors.

The next section will examine how and under what conditions states' expectations about both the distribution of power and the resolve of states converge. And this convergence is necessary in order to prevent preemptive attacks.

THEORETICAL ANALYSIS

Assume that two states in international system argue over strategic resources such as oil, coal, or territory. The total amount of resources is normalized as 1. In addition, let us assume that state 1 is a defending state, while state 2 is a challenging state.

Let π_1 be the expected utility of state 1, let p ($0 \leq p < 1$) be the probability of winning of state 1, let v_1 be the war value of state 1, and let c be the cost of war. To put it simply, I assume that if they win, they gain everything, while if they lose, they lose everything.¹⁷

$$\pi_1 = v_1 \cdot \{p \cdot 1 + (1 - p) \cdot 0\} - c \\ = p \cdot v_1 - c$$

In the same fashion, we can express the utility of state 2 as,

$$\pi_2 = q \cdot v_2 - c \quad (\text{where } p + q = 1) .$$

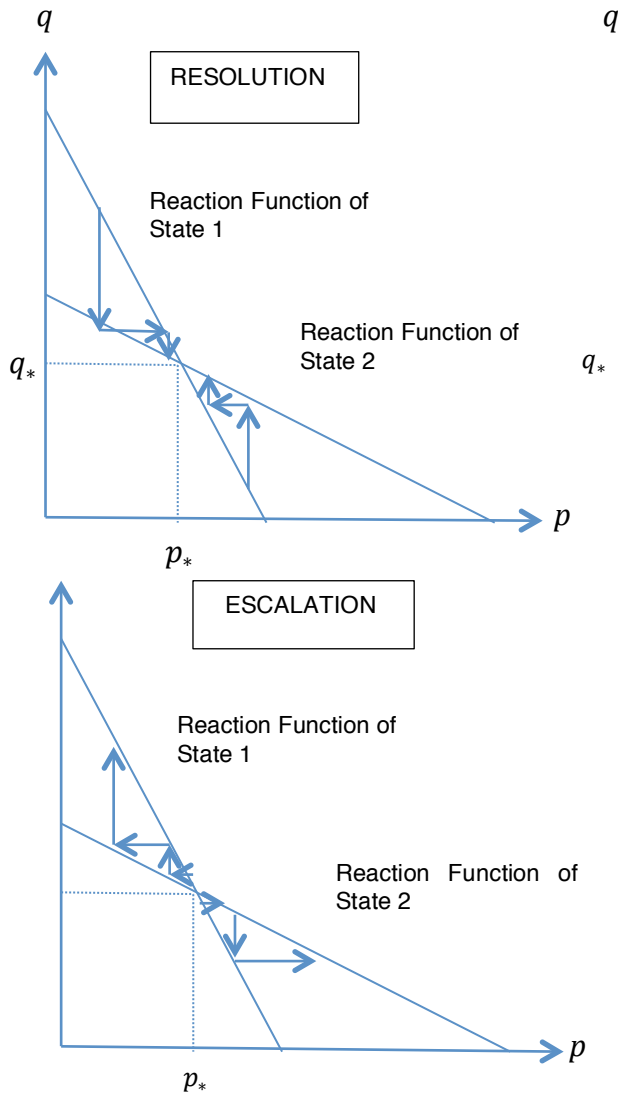
Note that p also stands for the balance of power in this international system or how resources are shared among states. This is because the distribution of resources among states is determined by the probability of winning wars.

¹⁵ Branislav L. Slantchev, "The Principle of Convergence in Wartime Negotiations," *American Political Science Review*, 97 (2003), 626.

¹⁶ *Ibid.* 621-32.

¹⁷ This model is consistent with those of Fearon (1997) and Wagner (2000) in the sense that it takes war to be a *costly lottery*. Harrison Wagner, "Bargaining and War," *American Journal of Political Science*, 44 (2000), 470.

Figure 1. Conflict Resolution and Escalation



As shown in Figure 1 above, I define the resolution of conflict as the convergence of beliefs about the probability of winning a war, while I define the escalation of conflict as the divergence of beliefs in that regard. The reason why we can define conflict as shown above is because if their beliefs converge, they have no incentive to change the status quo; if they diverge, they do have an incentive to change the status quo. The same rule is applicable to states' resolve, according to the explanation by Slantchev.

There is some latitude in states' actions within which their beliefs converge

and amount to peaceful resolution. The lower bound of this range is a signaling point, while its upper bound is a screening point, and they stand respectively for the minimum and maximum bounds necessary for convergence of belief. From here forward, I deduce those two points, which must be compatible to achieve their diplomatic purposes in this model.

Proposition 1

Let $\bar{p} = \frac{2}{5}(v_1 - 2v_2 + 2\alpha)$ (where α is a real number).

In the equilibrium, if $p > \bar{p}$, the conflict is resolved, while if $p < \bar{p}$, the conflict is escalated.

Proof: See Appendix.

This proposition has several implications. First, \bar{p} stands for the *signaling point*, meaning the minimum amount by which state 1 should demonstrate its resolve through activities such as increasing or deploying troops in order to make its message credible. If state 1 demonstrates resolve to less than this degree, other states will think that state 1 is not resolved and will be tempted to take advantage of it. Therefore, if state 1 seeks to make its signal credible, it should go at least to the degree \bar{p} , which leads to conflict resolution. This is because its belief in p converges with the equilibrium point, and no other states take action.

Proposition 2

Let $\underline{p} = \frac{1}{v_1 + v_2} \left(-\frac{1}{2}v_1^2 + \frac{1}{2}v_2^1 + 2v_1v_2 + v_2 - \beta \cdot v_2 \right)$.

In the equilibrium, if $p < \underline{p}$, the conflict is resolved, while if $p > \underline{p}$, the conflict is escalated.

If we let $A = v_1 + v_2$, we can get $\underline{p} = \frac{1}{2}A + \frac{-v_1^2 + (1-\beta)v_2}{A}$ (where β is a real number).

Proof: See Appendix

This proposition also has several implications. First, \underline{p} stands for the screening point, meaning the maximum degree by which state 1 should demonstrate its resolve in the effort to make its message credible. The reason for regarding the maximum point as the screening point is that it is necessary for a state to determine the maximum amount it can gain in searching for an opponent's resolve. If it crosses the limit, other states will get cautious and take some action against it. If state 1 wants to make its discourses credible, it should go at most to this point because here beliefs in \underline{p} converge.

Proposition 3

In the equilibrium, if $\bar{p} < p < \underline{p}$, the conflict is resolved; otherwise, it is escalated.

Proof: From the two propositions above, we can deduce that this proposition is inevitable.

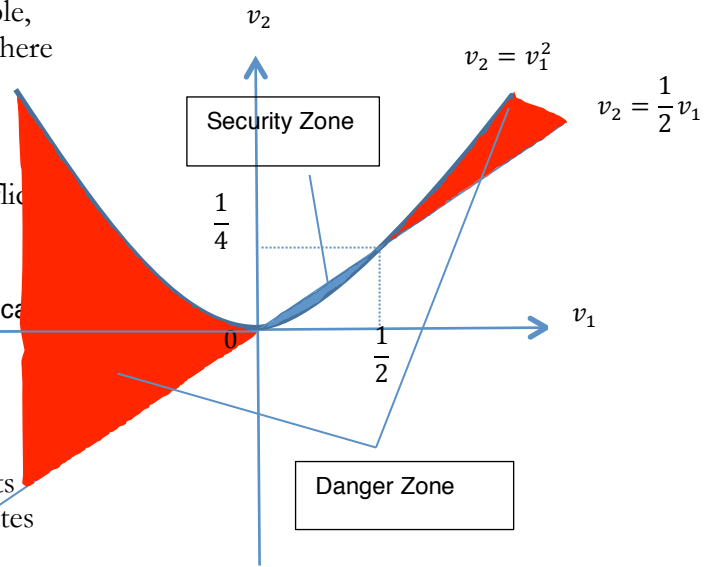
The most striking aspect of this proposition is that just by putting p at any point within this range, state 1 can make its messages credible and conflicts among states will be resolved. This is because any point within this range is Nash Equilibria: everyone will think that state 1 has no incentive to misrepresent its resolve within this range.

Now we can think about three cases. In the first case, the range between \bar{p} and \underline{p} expands. In this case, it becomes easier for states to put p within this range and to resolve conflicts. More formally, if $\bar{p} \equiv \frac{2}{5}(v_1 - 2v_2 + 2\alpha)$ decreases and $\underline{p} \equiv \frac{1}{2}A + \frac{-v_1^2 + (1-\beta)v_2}{A}$ increases, it becomes easier to converge expectations. I call this area the *security zone*.

The second case we will consider is one in which the range between \bar{p} and \underline{p} shrinks. In this case, it becomes more difficult for states to put p within this range and resolve conflicts. More formally, if $\bar{p} \equiv$

$\frac{2}{5}(v_1 - 2v_2 + 2\alpha)$ increases and $\underline{p} \equiv \frac{1}{2}A + \frac{-v_1^2 + (1-\beta)v_2}{A}$ decreases, it becomes more difficult to converge expectations. I call this area the *danger zone*. I represent these two areas graphically in Figure 2 below. However, I omit two variables, α and β , from the calculation for simplicity.

Figure 2. The Security Zone and the Danger Zone



Finally, let us consider a case in which \bar{p} goes ahead of \underline{p} and there is no room for states to take any measures to prevent war. More formally, if $\frac{2}{5}(v_1 - 2v_2 + 2\alpha) > \frac{1}{2}A + \frac{-v_1^2 + (1-\beta)v_2}{A}$, that is, if $v_1 < \frac{v_2 - \sqrt{2v_2(7v_2+5)}}{3}, \frac{v_2 + \sqrt{2v_2(7v_2+5)}}{3} < v_1$, the expectations of states never converge. This indicates that if the resolve of state 1 is either too much or too little, they have no choice but to go to war.

Based on this analysis, we can deduce several findings. The main finding is that war is more likely if the resolve of a challenging state exceeds half that of a defending state, while war is less likely if the resolve of a challenging state falls short of half that of a defending state. Note that $\frac{1}{2} : \frac{1}{4}$

equates to 2: 1, which represents the proportion of defending states and challenging ones.

In addition, war is less likely only when the resolve of both states is positive; and the resolve of defending states should be neither too high nor too low; paradoxically, if defending states are too resolved, they are significantly damaged as well.

TESTS OF THE THEORY

The theory above will be tested according to two criteria.¹⁸ First, what predictions can be inferred from the above theory? Second, how much history does this theory explain?

Predictions and Tests

The theory's predictions derived from its primary hypothesis. Namely, that war is more likely if the resolve of challenging states exceeds half that of defending states, while war is less likely if the resolve of challenging states falls short of half that of defending states. The following two predictions based on this theory are tested in a case study of the Gulf War of 1990-91. The outcomes vary sharply across time, creating a good setting for multiple within-case comparisons tests that contrast different periods within the same case.¹⁹

Defending states succeed in prevention of preemptive attacks if the resolve of challenging states' does not exceed the proportion.

Defending states fail in prevention of preemptive attacks if the resolve of challenging states' exceeds the proportion.

Test 1: Failure in Prevention

¹⁸ Stephen Van Evera, *Guide to Methods for Students of Political Science* (Ithaca, NY: Cornell University Press, 1997); Stephen Van Evera, "Offense, Defense, and the Causes of War," *International Security*, 22 (1998): 22.

¹⁹ Van Evera, *Guide to Methods for Students of Political Science*, 58-63.

Just before he decided to invade Kuwait, he assumed that the outcome of the Iran-Iraq war had propelled Iraq into the leadership of the Arab world in the post-Cold War era. This outcome imposed upon Iraq the duty to deter both Iranian hordes and Israeli adventurism, which spurred Iraq to reconstruct its economy and to expand its industrial-technological infrastructure. This could only be achieved by means of its oil revenues.²⁰

At the end of May 1990, during an Arab summit meeting convened in Baghdad, Saddam denounced the Arabs of the Gulf who were keeping the price of oil artificially low by producing beyond their OPEC quotas - to such an extent that the price in certain instances had plummeted to \$7 per barrel, although the agreed-upon price was \$18 per barrel-, and thereby engaging in economic sabotage of Iraq.²¹ In a memorandum dated July 15, addressed to the Secretary-General of the United Nations, Iraqi Foreign Minister Tariq Aziz explicitly named Kuwait and the United Arab Emirates as the two "culprits" in overproduction.²² To make matters worse, during the hostilities with Iran, Kuwait was drawing more than its allotment from the shared northern Rumaila oil field, whose southern tip straddled the border inland from the Gulf.²³

On July 16, at Saddam's request, Aziz sent a memorandum to Kuwait demanding \$2.4 billion in compensation for oil that Kuwait had pumped from the

²⁰ Walid Khalidi, "Iraq vs. Kuwait: Claims and Counterclaims" in *The Gulf War Reader*, ed. Micah Sifry and Christopher Cerf (Ithaca, NY: TimesBooks /Random House, 1991), 60-61.

²¹ Stein, "Deterrence and Compellence in the Gulf, 1990-91 -A Failed or Impossible Task?," 147-179; Khalidi, "Iraq vs. Kuwait", 63.

²² Bishara A. Bahbah, "The Crisis in the Gulf: - Why Iraq Invaded Kuwait" in Phyllis Bennis and Michel Moushabeck, eds., *Beyond The Storm: - A Gulf Crisis Reader* (New York: Olive Branch Press, 1991), 52-3; Khalidi, "Iraq vs. Kuwait," 63.

²³ Khalidi, "Iraq vs. Kuwait," 63.

disputed Rumaila oil field; \$12 billion in compensation for the depressed oil prices brought about by Kuwait's overproduction; forgiveness of Iraq's war debt of \$10 billion; and a lease on the strategic island of Bubiyan that controlled access to Iraq's only port, Umm Qasr.²⁴ Irrespective of the mediation efforts of King Fahd of Saudi Arabia, President Hosni Mubarak of Egypt, and King Hussein of Jordan, on July 24, two Iraqi armored divisions moved from their bases to positions on Kuwait's border.²⁵

While the crisis was intensified, Washington's signals to Iraq were ambiguous and contradictory.²⁶ On July 25, Saddam asked to meet with Ambassador April Glaspie within the hour, in response to the U. S. joint military exercises with the United Arab Emirates the previous day. Glaspie assured Saddam that the United States should express no opinion on the Kuwait issue, and that the issue was not associated with them. She then asked Saddam about the intention of Iraqi troop movements to the border with Kuwait. He informed her that he had just asked President Mubarak to assure Kuwait that they would do nothing until they held meetings. He added that if there were any hope in negotiations, nothing would happen, while if they were unable to find a solution, it would be natural for Iraq not to accept death.²⁷ This statement was designed to satisfy Ambassador

Glaspie. Her assumption was that the Iraqi tanks were meant to intimidate Kuwait at the negotiating table, and that they would probably succeed in doing so. She found it tolerable.²⁸

We can assume from the appearances that the resolve of Iraq was intensified day by day, while the United States was not resolved at all, which made war more and more likely. Saddam also thought that the United States would not wage war due to their Vietnam complex and the low tolerance of the American public to U.S. casualties.²⁹ Early in the morning on August 2, two Iraqi armored divisions spearheaded the attack against Kuwait.

Test 2: Success in Prevention

At the first full National Security Council (NSC) meeting after the Iraqi invasion of Kuwait on August 2, 1991, Schwarzkopf outlined two military options for the U.S. response: (1) punitive air strikes by carrier-based aircraft, and (2) a detailed existing contingency plan, Operation Plan (OP) 90-1002, which called for deploying more than four divisions and three aircraft carriers, for a total of 100,000 to 200,000 U.S. troops. The key divide in this August 3 meeting was between National Security Advisor Brent Scowcroft and Secretary of Defense Richard Cheney, who favored an expansive response, and Chairman of the Joint Chiefs of Staff General Colin Powell, who remained more cautious about committing U.S. military power and advocated the more limited goal of defending Saudi Arabia from further Iraqi aggression. In Powell's view, which became known as the "Powell Doctrine," the military was to be used as a last resort, and only then in an overwhelming fashion. By this time, however, President Bush had already begun to lean toward OP 90-1002, the "rollback" option

²⁴ Stein, "Deterrence and Compellence in the Gulf, 1990-91 –A Failed or Impossible Task?," 147-179.

²⁵ "Kuwait: How The West Blundered", in *The Gulf War Reader*, ed. Micah Sifry and Christopher Cerf (Ithaca, NY: TimesBooks /Random House, 1991), 104; Stein, "Deterrence and Compellence in the Gulf, 1990-91 –A Failed or Impossible Task?," 147-179.

²⁶ Stein, "Deterrence and Compellence in the Gulf, 1990-91 –A Failed or Impossible Task?," 147-179.

²⁷ "Kuwait: How The West Blundered", 104-5; Stein, "Deterrence and Compellence in the Gulf, 1990-91 –A Failed or Impossible Task?," 147-179.

²⁸ "Kuwait: How The West Blundered," 105.

²⁹ Bahbah, "The Crisis in the Gulf," 54.

over the more limited goal of “containment.”³⁰

The official Saudi “invitation” for U.S. military intervention came on August 7.³¹ The next day, August 8, a Defense Intelligence Agency officer informed Prince Bandar of what had happened during the Kuwait invasion and how Saddam was massing the same elite force of eight divisions on the Saudi border.³²

From then on, the U.S. was strengthening their signals to deter Iraq from invading Saudi Arabia, but at an incremental rate in order to reassure Iraq rather than encourage a preemptive attack. The first signal was sent on August 17 by U.S. officials including Defense Secretary Cheney. They intentionally leaked word to the Associated Press that the U.S. was preparing for a “long commitment” in the Gulf.³³ They also leaked the real number of troops up to 250,000 to the press.³⁴ The next signal was the Pentagon’s exponentially inflated troop projections needed to “defend” Saudi Arabia against an Iraqi attack. On August 22, the president called up an initial 40,000 members of military reserve units, and by September 6 the number of troops in Saudi Arabia had reached 100,000. On September 15, U.S. officials announced they had established a credible defense and were now building up their offensive capability.³⁵ Finally, on September 21, U.S. intelligence claimed that Saddam’s forces were digging in, moving into even more defensive positions. This made an

offensive attack by Saddam into Saudi Arabia less likely.³⁶

In those days, when air power was not as well recognized, it was ground troops more than airstrikes that showed the governments’ resolve. Saddam had moved his eight divisions, an estimated 100,000 men, to the Saudi border while the United States went public with the concrete number of troops who would be sent to defend Saudi Arabia and then put the plan into practice. What is important in this study is the fact that when Saddam recognized that the United States put twice as many troops as his on the Saudi border, he gave up invading Saudi Arabia. This corroborates the prediction that defending states succeed in both deterrence and reassurance when their resolve exceeds twice that of challenging states’.

CONCLUSION

This article argues that war is more likely when the resolve of challenging states exceeds twice that of defending states. With respect to the causes of war, states cannot achieve both diplomatic policies, namely, deterrence and reassurance, at the same time unless they combine the techniques of signaling and screening.

Based on the assumption that war can be represented as the divergence of state beliefs about the probability of winning and state resolve, I establish the minimum and maximum points necessary for their beliefs to converge, which can be defined as signaling and screening points. I then assume that if those ranges expand, it becomes easy for states to determine the points that are necessary for the settling of disputes, and vice versa. This enables me to present the hypotheses mentioned above. I deduce several predictions from those hypotheses, which are tested with the case of the Gulf War of 1990-91, which is rich in both political success and failure. I then

³⁰ Jonathan Monten and Andrew Bennett, “Models of Crisis Decision Making and the 1990-91 Gulf War,” *International Security*, 19 (2010):498-503.

³¹ Steve Niva, “The Battle is Joined,” in *Beyond The Storm: – A Gulf Crisis Reader*, ed. Michel Moushabeck and Phyllis Bennis (New York: Olive Branch Press, 1991), 57.

³² Bob Woodward, *The Commanders* (New York: Simon and Schuster, 1991), 278.

³³ Niva, “The Battle is Joined,” 59.

³⁴ Woodward, *The Commanders*, 297.

³⁵ Niva, “The Battle is Joined,” 59.

³⁶ Woodward, *The Commanders*, 297.

demonstrate that these predictions correlated with what happened in the Gulf.

This study leaves many themes of remaining research. The first topic we need to tackle is how to deal with the influence of state growth, which is categorized as a commitment problem. In order to complete the research, we need to consider a dynamic version of this model by introducing new variables such as the growth rates of states. Some previous research has already tackled this problem with distinguished implications.³⁷ What we need to do next is to combine these achievements with the findings of this research in order to improve the ways in which states can achieve their diplomatic purposes, even when there is a great disparity in growth rates among states.

APPENDIX

Proof of Proposition 1

Consider the Cournot Equilibrium. The reason why I consider the Cournot Equilibrium is because in equilibrium both actors' beliefs about the probability of winning war are supposed to converge. Thus, I start with both actors' reaction functions so that their beliefs will converge as a result of their profit maximizations.

Assume that state 1 is a defender, and its reaction function is $q = p$, while state 2 is a challenger, and its reaction function is $q = a \cdot p + \alpha$ (where α is a real number). In this case, the condition of equilibrium at point t is $p_t = a \cdot p_{t-1} + \alpha$. In addition, in equilibrium, $p_* = a \cdot p_* + \alpha$.

If we assume that $\Delta p_t \equiv p_t - p_*$, we can get $\Delta p_t = a \Delta p_{t-1} = \dots = a^t \Delta p_0$. (where a is a real number). The important point in this

equilibrium is that the condition of conflict resolution is $|a| < 1$.

Because the reaction function of state 1 is $q = p$, we can get $\frac{\partial \pi_1}{\partial p} = p - q = 0$.

Therefore, $\pi_1 = \frac{1}{2}p^2 - q \cdot p + \beta = p \cdot \left(\frac{1}{2}p - q\right) + \beta$ (where β is a real number).

Since we assume $\pi_1 = p \cdot v_1 - c$, we can get $v_1 = \frac{1}{2}p - q$. Moreover, since $p + q = 1$, we can get $1 = \frac{3}{2}p - v_1 \dots \ominus$.

On the other hand, because the reaction function of state 2 is $q = a \cdot p + \alpha$, we can get $\frac{\partial \pi_2}{\partial q} = ap + \alpha - q = 0$.

Therefore, $\pi_2 = -\frac{1}{2}q^2 + (ap + \alpha)q + \gamma = q \left\{-\frac{1}{2}q + (ap + \alpha)\right\} + \gamma$ (where γ is a real number).

Since we assume $\pi_2 = q \cdot v_2 - c$, we can get $v_2 = -\frac{1}{2}q + (ap + \alpha)$.

Now, since $p + q = 1$, we can get $1 = (2a + 1)p - 2v_2 + 2\alpha \dots \ominus$.

Because \ominus and \ominus should be compatible, $\frac{3}{2}p - v_1 = (2a + 1)p - 2v_2 + 2\alpha$.

Therefore we can get $a = \frac{(-v_1 + 2v_2 - 2\alpha)}{2p} + 1/4$.

When $a \geq 0$, the condition of $|a| < 1$ is $\frac{(-v_1 + 2v_2 - 2\alpha)}{2p} + 1/4 < 1$.

Therefore, we can get $p > \frac{2}{3}(-v_1 + 2v_2 - 2\alpha)$.

When $a < 0$, the condition of $|a| < 1$ is $\frac{-(-v_1 + 2v_2 - 2\alpha)}{2p} - 1/4 < 1$.

Therefore, we can get $p > \frac{2}{5}(v_1 - 2v_2 + 2\alpha)$. Q.E.D.

Proof of Proposition 2

Consider the Bertrand Equilibrium. The reason why I consider the Bertrand Equilibrium is because in equilibrium both actors' beliefs about the war values are supposed to converge.

³⁷ Powell, "The Inefficient Use of Power: Costly Conflict with Complete Information," *American Political Science Review*, 98 (2004): 231-41; Robert Powell, "War as a Commitment Problem," *International Organization*, 60 (2006): 169-203.

Thus, I start with both actors' reaction functions so that their beliefs will converge as a result of their profit maximizations.

Assume that state 1 is a defender and its reaction function is $v_2 = v_1$, while state 2 is a challenger and its reaction function is $v_2 = a \cdot v_1 + \beta$ (where β is a real number). The equilibrium at point t is $v_{1t} = a \cdot v_{1t-1} + \beta$, while in equilibrium, $v_{1*} = a \cdot v_{1*} + \beta$.

If we assume that $\Delta v_{1t} \equiv v_{1t} - v_{1*}$, we can get $\Delta v_{1t} = a \Delta v_{1t-1} = \dots = a^t \Delta v_{10}$ (where a is a real number). The important point in this equilibrium is that the condition of conflict resolution is $|a| < 1$.

Because the reaction function of state 1 is

$$v_2 = v_1, \text{ we can get } \frac{\partial \pi_1}{\partial v_1} = v_2 - v_1 = 0.$$

$$\text{Therefore, } \pi_1 = -\frac{1}{2} v_1^2 + v_2 \cdot v_1 + \gamma = v_1 \cdot \left(-\frac{1}{2} v_1 + v_2\right) + \gamma (\gamma \text{ is a real number}).$$

Since we assume $\pi_1 = p \cdot v_1 - c$, we can get

$$p = -\frac{1}{2} v_1 + v_2.$$

Because $S = p \cdot v_1 + q \cdot v_2$, we can get

$$S = -\frac{1}{2} v_2^2 + v_1 \cdot v_2 + q \cdot v_2 \cdot \dots \ominus$$

(I assume that S stands for the supply of public goods provided by those states).

On the other hand, the expected utility of state 2 is $v_2 = a \cdot v_1 + \beta$; hence, we can get

$$\frac{\partial \pi_2}{\partial v_2} = a \cdot v_1 + \beta - v_2 = 0. \text{ Therefore, } \pi_2 =$$

$$-\frac{1}{2} v_2^2 + (a \cdot v_1 + \beta) \cdot v_2 + \delta =$$

$$v_2 \left\{ -\frac{1}{2} v_2 + (a \cdot v_1 + \beta) \right\} + \delta \text{ (where } \delta \text{ is a real number).}$$

Since we assume $\pi_2 = q \cdot v_2 - c$, we can get

$$q = -\frac{1}{2} v_2 + (a \cdot v_1 + \beta).$$

Because $S = p \cdot v_1 + q \cdot v_2$, we can get

$$S = -\frac{1}{2} v_2^2 + (a \cdot v_1 + \beta) v_2 + p \cdot v_1 \cdot \dots \ominus$$

Now, both \ominus and \ominus should be compatible,

$$\text{such that } -\frac{1}{2} v_2^2 + v_1 v_2 + q \cdot v_2 =$$

$$-\frac{1}{2} v_2^2 + (a \cdot v_1 + \beta) v_2 + p \cdot v_1.$$

$$\text{Therefore, we can get } a = \frac{1}{2} \left(\frac{v_2}{v_1} - \frac{v_1}{v_2} \right) - p \cdot$$

$$\frac{1}{v_2} + q \cdot \frac{1}{v_1} - \beta \cdot \frac{1}{v_1} + 1.$$

When $a \geq 0$, the condition of $|a| < 1$ is

$$\frac{1}{2} \left(\frac{v_2}{v_1} - \frac{v_1}{v_2} \right) - p \cdot \frac{1}{v_2} + q \cdot \frac{1}{v_1} - \beta \cdot \frac{1}{v_1} + 1 < 1.$$

$$\text{Therefore, we can get } p > -\frac{1}{2} \cdot v_1 + \frac{v_2}{v_1} \cdot$$

$$\left(\frac{1}{2} v_2 - q - \beta \right).$$

When $a < 0$, the condition of $|a| < 1$ is

$$-\frac{1}{2} \left(\frac{v_2}{v_1} - \frac{v_1}{v_2} \right) + p \cdot \frac{1}{v_2} - q \cdot \frac{1}{v_1} + \beta \cdot \frac{1}{v_1} - 1 <$$

$$1. \text{ Therefore, we can get } p < \frac{1}{v_1 + v_2} \left(-\frac{1}{2} v_1^2 + \right.$$

$$\left. \frac{1}{2} v_2^2 + 2v_1 v_2 + v_2 - \beta \cdot v_2 \right).$$

Since we need to find the upper bound of p ,

we should assume that $a < 0$.

Therefore, we should also assume that $a < 0$ in proposition 1.Q.E.D.